

# Model Predictions for Neutrino Oscillation Parameters

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## Abstract

We have surveyed leptonic and grand unified models of neutrino masses and mixings in the literature which are still viable and give numerical predictions for the reactor angle,  $\theta_{13}$ . The results are of considerable interest in anticipation of the next generation reactor experiments and the possible future need for neutrino factories. Of the 63 models considered which were published or posted on the Archive before June 2006, half predict values of  $\sin^2 2\theta_{13} \gtrsim 0.015$ , which should yield positive signals for  $\bar{\nu}_e$  disappearance in the reactor experiments planned for the near future. Depending upon the outcome of those experiments, half of the models can be eliminated on the basis of the presence or absence of such an observed  $\bar{\nu}_e$  disappearance signal.

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## I. INTRODUCTION

With the confirmation of atmospheric muon-neutrino oscillations by the Super-Kamiokande collaboration [1], the need for physics beyond the standard model became clear. This prompted many authors to construct mass matrix models to explain the neutrino mixings and the still not well-determined neutrino mass spectrum. Meanwhile underground and reactor experiments [2] confirmed the existence of solar electron-neutrino oscillations as a solution to the solar neutrino puzzle, whereby the detection of solar electron-type neutrinos was depleted [3] relative to the standard solar model projections [4]. Since then many of the first round of models have fallen by the wayside, as the once-preferred small mixing angle (SMA) solution for the solar neutrino oscillations has been replaced by the large mixing angle (LMA) solution. Accelerator experiments have now also contributed to the greatly increased precision of all observed oscillation results. In particular, the observed large atmospheric neutrino mixing has persisted as a nearly maximal  $\nu_\mu - \nu_\tau$  mixing [5]. The solar neutrino mixing is now known to be large but not maximal [6]. On the other hand, only a relatively small upper limit has been placed on the  $\bar{\nu}_e$  mixing with the other two antineutrinos by two reactor neutrino experiments carried out several years ago [7]. Clearly the neutrino mixing pattern is totally unlike that observed in the quark sector.

Despite the refinement of the experimental results, many of the more recent neutrino models have survived to date due to the uncertainty in the precise magnitude for the reactor angle,  $\theta_{13}$ , and the unknown mass hierarchy for the neutrino spectrum: normal, degenerate, or inverted. In this paper we are primarily concerned with the model predictions for the reactor angle. While new reactor experiments [8] are being planned or are already in construction to measure this angle down to  $3^\circ$ , i.e.,  $\sin^2 2\theta_{13} \simeq 0.01$ , it is of considerable interest to learn whether that reach will be sufficient to determine the angle through the detection of a  $\bar{\nu}_e$  disappearance signal, or whether a neutrino factory will be required which can probe a much smaller angle [9]. For this purpose, we have surveyed 63 models in the literature which are still viable candidates and have reasonably well-defined predictions for  $\theta_{13}$ . Roughly half of the models predict that  $\sin^2 2\theta_{13}$  covers the range from 0.015 to the present upper bound of 0.15 [7]. Hence half of the models can be eliminated in the next round of reactor experiments, based on the presence or absence of an observed  $\bar{\nu}_e$  disappearance signal. While none of the models proposed so far may be correct, the distribution of results for  $\sin^2 2\theta_{13}$  does provide some indication of what one may expect to find.

In Sect. II we define the mixing angles and state the present experimental results. A brief description of the types of models based on lepton flavor symmetries and/or grand unification

for the quarks and leptons is presented in Sect. III. Several more comprehensive reviews of mass matrix models exist in the literature [10], and we refer the interested reader to them for additional details. We have restricted our attention to models with just three active neutrinos and no sterile neutrinos. This seems justified at present in light of the conflicting evidence for an additional oscillation between  $\bar{\nu}_\mu$  and  $\bar{\nu}_e$  observed by the LSND Collaboration [11], but not by the Karmen Collaboration [12]. The MiniBooNE Collaboration is expected to throw more light on this situation shortly [13]. We make clear our acceptance criteria for the three flavor neutrino models to be considered and give tables of the mixing angles predicted for each in Sect. IV. Histograms for  $\sin^2 \theta_{13}$  are plotted for all 63 models together, for the 34 models which give numerical predictions for all three mixing angles, and separately for those models with normal or inverted neutrino mass hierarchy. Our conclusions are presented in Sect. V.

## II. LEPTONIC MIXING MATRIX AND PRESENT EXPERIMENTAL INFORMATION

Here for completeness we present a brief description of the well-known neutrino mixing matrix and give an up-to-date summary of the constraints on the mixing angles.

The light left-handed neutrino flavor states are related to the neutrino mass eigenstates by the linear combinations

$$|\nu_\alpha\rangle = \sum_i (U_{\nu_L})_{\alpha i} |\nu_i\rangle, \quad (1)$$

where  $\alpha = e, \mu, \tau$  and  $i = 1, 2, 3$ . If neutrinos are Majorana particles, the  $U_{\nu_L}$  transformation matrix is obtained by diagonalization of the effective left-handed Majorana neutrino mass matrix according to

$$M_\nu^{diag} = U_{\nu_L}^T M_\nu U_{\nu_L}, \quad (2)$$

where  $M_\nu$  is model-dependent and is typically constructed from some basic symmetry principle, or follows from the seesaw mechanism [14] in grand unified (GUT) models. For models in which the light neutrinos are assumed to be Dirac particles, the  $U_{\nu_L}$  unitary transformation matrix can be obtained from the bi-unitary transformation which relates the flavor basis to the mass basis as in

$$M_\nu^{diag} = U_{\nu_R}^\dagger M_\nu^D U_{\nu_L}. \quad (3)$$

The light Dirac neutrino mass matrix,  $M_\nu^D$ , is also constructed according to some symmetry principle.

In models based on some assumed lepton flavor symmetry, the charged lepton mass matrix is assumed to be diagonal in the lepton flavor basis with the charged lepton masses  $m_e$ ,  $m_\mu$  and  $m_\tau$  ordered along the diagonal; thus  $U_{LL}$ , the counterpart of  $U_{\nu L}$ , is just the identity matrix. For GUT models, on the other hand, the charged lepton mass matrix is generally not diagonal in the GUT flavor basis. In this case in analogy with Eq. (3) above,  $U_{LL}$  can be determined from the bi-unitary transformation

$$M_L^{diag} = U_{LR}^\dagger M_L U_{LL}, \quad (4)$$

where we have again adopted the convention that the right-handed fields act on the left and the left-handed fields on the right of the Dirac mass matrix. For either type of model, the left-handed neutrino Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix is then given by [15]

$$V_{PMNS} \equiv U_{LL}^\dagger U_{\nu L} = U_{PMNS} \Phi. \quad (5)$$

It is convenient to choose by convention

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (6)$$

in analogy with the quark mixing matrix, along with the Majorana phase matrix,

$$\Phi = \text{diag}(e^{i\chi_1}, e^{i\chi_2}, 1), \quad (7)$$

in terms of the three mixing angles,  $\theta_{12}$ ,  $\theta_{23}$  and  $\theta_{13}$ ; the Dirac  $CP$  phase,  $\delta$ ; and the two Majorana phases,  $\chi_1$  and  $\chi_2$ . With Dirac neutrinos, one is free to make phase transformations on both  $U_{LL}$  and  $U_{\nu L}$ , so  $\Phi$  is just the identity matrix. But in the Majorana neutrino case where  $U_{\nu L}$  is defined by Eq. (2), an arbitrary phase transformation is not possible when one demands real diagonal neutrino mass entries. Hence the presence of the Majorana phase matrix is required in order to adopt the convention for  $U_{PMNS}$  specified in Eq. (6).

We now summarize the numerical information for these mixing parameters, as given by Maltoni, Schwetz, Tortola, and Valle [16] in a recent updated global analysis which incorporates all the latest results cited in [3]-[7]. Within  $2\sigma$  accuracy, they found

$$\begin{aligned} \Delta m_{21}^2 &= (7.3 - 8.5) \times 10^{-5} \text{ eV}^2, \\ \Delta m_{31}^2 &= (2.2 - 3.0) \times 10^{-3} \text{ eV}^2, \\ \sin^2 \theta_{12} &= 0.26 - 0.36, \\ \sin^2 \theta_{23} &= 0.38 - 0.63, \\ \sin^2 \theta_{13} &\leq 0.025. \end{aligned} \quad (8)$$

No information exists for the Dirac or Majorana  $CP$  phases, for the neutrino mass hierarchy, or for the Dirac *vs.* Majorana nature of the neutrinos. Of special interest to us is the upper bound on  $\sin^2 \theta_{13}$  which has mainly been determined by a non-observation of a depletion of the  $\bar{\nu}_e$  flux from the CHOOZ reactor [7]. In Sect. IV we shall turn to the model selection criterion we have used to extract the various model predictions. But first we give descriptions of the types of models that have been proposed.

### III. DESCRIPTIONS OF MODELS

Most neutrino mass matrix models fall into two broad classes: those based on a lepton flavor symmetry which applies only to the charged lepton and light neutrino mass matrices and those based on some grand unification scheme which applies to both leptons and quarks. The latter class typically involves some family unification group which unites the quarks and leptons at the high gauge unification scale, while a flavor symmetry applying to the corresponding members of different families may or may not be invoked. Moreover, right-handed singlet neutrinos belong to irreducible representations and provide an exquisite way to give ultralight masses via the seesaw mechanism [14] to the observed left-handed neutrinos. In the purely leptonic models some lepton flavor symmetry is generally considered, but the ultralight  $0.001 - 0.1$  eV neutrino mass scales typically remain a puzzle. Several symmetry schemes have been proposed within each of the two classes. Exceptional cases involve a model based on anarchy with no flavor symmetry and models with sequential right-handed neutrino dominance but no unification group identified.

Of considerable interest is whether a given model exhibits a normal or inverted mass hierarchy. Models of either hierarchy can be found in several categories, while for others only one or the other is definitely preferred. We shall give broad and general descriptions here for each type of model considered. As cited earlier, we refer the reader to the comprehensive reviews [10] for more details.

#### A. Anarchy – a Model of Flavor with No Flavor Symmetry

Contrary to our general study of mixing predictions of neutrino mass matrix models with some specified symmetry, we first consider a model based on neutrino mass anarchy, where the neutrino mass matrix is completely random as no flavor symmetry is specified [17]. With this assumption of no fundamental distinction among the three flavors of neutrinos and hence no preferred basis for the neutrino states, one can simply examine statistically the neutrino mixing matrix. Initially

the authors of [17] first used a Monte Carlo analysis to test how many sets of mixing angles passed imposed cuts and then later applied the invariant Haar measure to the mixing angles. Still later de Gouvea and Murayama [18] refined the analysis by applying the Kolmogorov-Smirnov statistical test to the single  $\sin^2 \theta_{13}$  variable to obtain a more precise lower bound on its expected value which is quoted in the first table.

## B. Models with Lepton Flavor Symmetries

It is customary for models of this type to be formulated in the leptonic flavor basis for which the charged lepton mass matrix is diagonal. The differentiating feature of these models then resides solely in the light left-handed neutrino mass matrix.

### 1. $\mu - \tau$ Interchange Symmetry and $L_e - L_\mu - L_\tau$ Conserved Flavor Symmetry

The most general neutrino mass matrix exhibiting a  $\mu - \tau$  interchange symmetry is given by [19]

$$M_\nu = \begin{pmatrix} a & b & b \\ b & c & d \\ b & d & c \end{pmatrix}. \quad (9)$$

If the symmetry is unbroken and  $a, b < c = d$ , a normal mass hierarchy is obtained with  $\Delta m_{32}^2 = m_3^2$ ,  $\Delta m_{21}^2 = 0$ ,  $\sin^2 2\theta_{23} = 1$ , and  $\sin^2 2\theta_{12} = \sin^2 2\theta_{13} = 0$ , as the atmospheric neutrino mixing is maximal while the solar and reactor neutrino mixings vanish. On the other hand, for the simple form  $a = c = d = 0$ , the symmetry remains unbroken and an inverted mass hierarchy is obtained. In this case  $\Delta m_{32}^2 = \Delta m_{31}^2 = m_1^2$ ,  $\Delta m_{12}^2 = 0$ , and  $\sin^2 2\theta_{23} = \sin^2 2\theta_{12} = 1$  with  $\sin^2 2\theta_{13} = 0$  as both the atmospheric and solar neutrino mixings are maximal, while the reactor neutrino mixing vanishes. Neither of these sets of predictions are observed experimentally, so soft symmetry breaking of either matrix texture must be introduced to obtain an acceptable model with the initial hierarchy unaltered.

The special inverted hierarchy texture case cited above actually exhibits an enhanced  $L_e - L_\mu - L_\tau$  symmetry which is more generally of the form [19]

$$M_\nu = \begin{pmatrix} 0 & b & b' \\ b & 0 & 0 \\ b' & 0 & 0 \end{pmatrix}. \quad (10)$$

Independent of the relative magnitudes of  $b$  and  $b'$ , this rank-2 matrix leads to a neutrino mass hierarchy which is inverted. Again soft symmetry breaking must be introduced in order for the model to be experimentally viable. Examples of these two lepton flavor symmetries are grouped together in the tables presented.

## 2. $S_3$ Lepton Flavor Symmetry

In the case of  $S_3$  lepton flavor symmetry involving the permutation group of three flavors applied to both rows and columns of the neutrino mass matrix, the most general texture is [20]

$$M_\nu = \begin{pmatrix} a & b & b \\ b & a & b \\ b & b & a \end{pmatrix}, \quad (11)$$

in terms of two independent parameters,  $a$  and  $b$ . Alternatively, one can consider as a basis in the flavor space the unit mass matrix with 1's down the diagonal and the democratic mass matrix with all unit elements. For the unit matrix, the mass spectrum is clearly degenerate and all mixing angles vanish. On the other hand, the rank-1 democratic matrix yields a normal mass hierarchy; moreover, the neutrino mixing matrix has the tribimaximal form suggested by Harrison, Perkins, and Scott [21]:

$$U_{PMNS} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix}, \quad (12)$$

corresponding to the mixing relations  $\sin^2 2\theta_{23} = 1$ ,  $\sin^2 \theta_{12} = 1/3$ , and  $\sin^2 \theta_{13} = 0$ . These results are close to their experimental values, so only small corrections may be required. However, the diagonal charged lepton mass matrix clearly does not obey the  $S_3$  symmetry being considered.

## 3. $A_4$ Lepton Flavor Symmetry

The permutation group  $S_4$  of four objects was first considered by Ma and Rajasekaran [22] as a discrete flavor symmetry. Its non-Abelian subgroups are  $S_3$ ,  $D_4$  and  $A_4$ . What makes the subgroup  $A_4$  of the twelve even permutations of  $S_4$  of particular interest is the fact that it is also the smallest discrete subgroup of  $SO(3)$  which has at least one three-dimensional representation. In fact, there are just four irreducible representations, one triplet and three singlets. The three lepton doublets

can be placed in the  $\mathbf{3}$  while the three right-handed charged leptons are each placed in one of the singlets of  $A_4$ . With three Higgs doublets also transforming as a triplet, one can construct the charged lepton mass matrix. On the other hand, if the right-handed neutrinos are placed in a triplet representation, the Dirac and Majorana neutrino mass matrices can be generated with Higgs singlets. This application of  $A_4$  and other variations show that tribimaximal mixing of neutrinos can also be achieved while alleviating some of the problems of the  $S_3$  flavor symmetry.

#### 4. Other Lepton Flavor Symmetries

Other lepton flavor symmetries have also been considered in the literature including, for example,  $SO(3)$  and  $SU(3)$ . Another very popular starting point involves the arbitrary assignment of texture zeros in the light neutrino mass matrix [23]. By doing so, one eliminates those assignments which do not yield a neutrino mixing matrix mimicing the nearly tribimaximal mixing form. The positions of the texture zeros may then point the way to some underlying flavor symmetry. Some of these models are included in our study.

Various attempts have also been made to extend the lepton flavor symmetries proposed to the quark sector. But without a grand unification symmetry framework, they have met with rather mixed results.

### C. Sequential Right-Handed Neutrino Dominance

Another class of models which do not neatly fit into the lepton flavor symmetry class or the grand unification class are models with right-handed neutrino dominance [24]. Here three right-handed neutrinos are introduced which have a strong hierarchical mass spectrum. In the absence of any family gauge symmetry, one can still deduce that the light neutrino mass spectrum is controlled by the sequential dominance of the right-handed neutrinos, i.e., the mass of the heaviest left-handed neutrino is determined largely by the mass of the lightest right-handed neutrino, etc. With this type of model, authors have shown that near tribimaximal mixings can also be obtained.

### D. Grand Unified Models

An alternative approach is to start with a vertical family unification symmetry at some grand unification scale [25]. One can then try to impose a flavor symmetry on this structure which relates the corresponding members of each family; however, in many cases, one simply adopts



an effective operator approach which arbitrarily assigns certain operators to each element of the mass matrices. Models formulated in this framework are obviously much more ambitious than the previous ones dealing only with the lepton sector. The mass and mixing results for the leptons are highly constrained by the input parameters introduced for the quark sector, since the Yukawa couplings apply to both quark and lepton Dirac mass matrices. Nevertheless, successful models exist in the literature which are still viable, even after tighter constraints on both the quark and lepton mixing parameters have been obtained experimentally. We briefly discuss models in the following categories.

### 1. $SU(5)$ and Flipped $SU(5)$ Models

Grand unification with a high scale  $SU(5)$  symmetry was first proposed by Georgi and Glashow [26], where the quarks, leptons and left-handed neutrinos can be placed into **10** and  $\bar{\mathbf{5}}$  representations. With the appearance of neutrino oscillations it was then suggested to place the right-handed neutrinos into  $SU(5)$  singlets. The Higgs fields are conventionally placed in the  $\bar{\mathbf{5}}$  and **24** representations. This group symmetry with a minimal Higgs structure proved to be less interesting when the limit on the proton decay lifetime increased several orders of magnitude above the predicted range of  $10^{29} - 10^{30}$  years.

An alternative procedure is to consider flipped  $SU(5)$  [27] in which the charged lepton and right-handed neutrino are interchanged in the **10** and **1** representations, as well as the conjugate up and conjugate down quarks in the **10** and  $\bar{\mathbf{5}}$  representations with respect to the usual  $SU(5)$  assignments. While models of this type have been pursued in the literature, none of them have numerical predictions for the reactor angle  $\theta_{13}$ , so we do not elaborate on them further.

### 2. $SO(10)$ Models with Higgs in High Rank Representations

The  $SO(10)$  grand unification symmetry is an economical and attractive one [25], for all sixteen left-handed quark and lepton fields and their left-handed conjugates fit neatly into one **16** representation per family. Many models exist in the literature which differ from one another by their Higgs representation assignments and flavor symmetry imposed, if any. To appreciate this, it is of interest to note the following decompositions of the direct product of representations:

$$\begin{aligned}\mathbf{16} \otimes \mathbf{16} &= \mathbf{10}_s \oplus \mathbf{120}_a \oplus \mathbf{126}_s, \\ \mathbf{16} \otimes \bar{\mathbf{16}} &= \mathbf{1} \oplus \mathbf{45} \oplus \mathbf{210},\end{aligned}\tag{13}$$

where in the first product the  $\mathbf{10}$  and  $\mathbf{126}$  matrices are symmetric, while the  $\mathbf{120}$  is antisymmetric.

For the simplest Higgs structure [28] one can assume the presence of one  $\mathbf{10}_H$ , one  $\overline{\mathbf{126}}_H$ , and one  $\mathbf{126}_H$  representations, where the latter is needed to preserve a D-flat direction at the GUT scale in supersymmetric models. The  $\mathbf{10}_H$  contains two Higgs doublets, which appear in the  $\mathbf{5} + \overline{\mathbf{5}}$   $SU(5)$  decomposition, and contribute to the Dirac mass matrices, while the  $\overline{\mathbf{126}}_H$  contains a Higgs singlet which contributes to the right-handed Majorana neutrino mass matrix. Recently additional Higgs fields in the  $\mathbf{120}_H$ ,  $\mathbf{210}_H$ , and  $\mathbf{45}_H$  representations have also been considered by model builders in order to fine tune their predictions. The Dirac mass matrices will then have symmetric or antisymmetric textures, or even matrix elements with linear combinations of the two forms. The high ranks of these Higgs representations, rank-3, 4, and 5 are somewhat disfavored in the string theory framework [29].

With the vacuum expectation value of the  $\overline{\mathbf{126}}_H$  appearing near the GUT scale and giving massive entries to the right-handed Majorana mass matrix, the conventional type I seesaw mechanism [14] provides a ready explanation for the ultralight left-handed neutrinos:

$$M_\nu = -M_N^T M_R^{-1} M_N, \quad (14)$$

where  $M_N$  is the Dirac neutrino mass matrix and  $M_R$  is the right-handed Majorana matrix in the basis convention that the conjugate left-handed fields appear on the left and the left-handed fields on the right of  $M_N$ . It is easy to show that models involving a type I seesaw yield a normal mass hierarchy for the light neutrinos [31].

Some authors also allow the possibility that Higgs triplet VEV's exist in the  $\overline{\mathbf{126}}_H$  and can give non-zero entries to the left-handed Majorana mass matrix,  $M_{LL}$ . One then generates the light left-handed neutrinos through the type II (or mixed) seesaw mechanism [30]:

$$M_\nu = M_{LL} - M_N^T M_R^{-1} M_N. \quad (15)$$

In this case, the light neutrino mass hierarchy can be normal, degenerate, or inverted, the latter two occurring when the  $M_{LL}$  contributions are comparable to or larger than the type I contributions.

One possible breaking pattern of  $SO(10)$  down to the SM gauge group has the Pati-Salam group [32],  $SU(4) \times SU(2)_L \times SU(2)_R$ , as the intermediate gauge group,

$$SO(10) \rightarrow SU(4) \times SU(2)_L \times SU(2)_R \rightarrow SU(3) \times SU(2)_L \times U(1)_Y \quad (16)$$

This breaking can be achieved with a minimal Higgs content that has one  $\mathbf{10}_H$ , one  $\mathbf{45}_H$ , one  $\mathbf{54}_H$  and a conjugate pair of  $\mathbf{126}_H \oplus \overline{\mathbf{126}}_H$ . Due to the left-right symmetry, the resulting mass

matrices are symmetric. In addition, if the minimal Higgs content described above is utilized, one has the following relations: the up type quark mass matrix and the Dirac neutrino mass matrix are identical, while the mass matrix of the down type quarks and that of the charged leptons are identical, up to some calculable Clebsch-Gordan coefficients which, when combined with the family symmetry, can be used to obtain the Georgi-Jarlskog relations [33] required by phenomenology. The Majorana mass terms for the right-handed neutrinos arise from coupling to the  $\overline{\mathbf{126}}_H$ . These intra-family relations among the mass matrices greatly reduce the number of free parameters in the Yukawa sector, making these models very predictive.

### 3. $SO(10)$ Models with Lopsided Mass Matrices

While the minimal Higgs models discussed above naturally preserve R-parity when the  $\overline{\mathbf{126}}_H$  develops a VEV and lepton number is violated by two units, they suffer from the disadvantage that they become non-perturbative above the GUT scale due to the high rank of the representations. Models which do not share this problem can be constructed by using lower rank Higgs representations [34] including:  $\mathbf{10}_H$ 's, one  $\mathbf{45}_H$ , and one or two pairs of  $\mathbf{16}_H$  -  $\overline{\mathbf{16}}_H$ . At the GUT scale, VEV's of the  $\mathbf{45}$  and the  $SU(5)$  singlet parts of a  $\mathbf{16}_H$  -  $\overline{\mathbf{16}}_H$  pair break the  $SO(10)$  symmetry to that of the standard model. Near GUT scale masses are generated for the right-handed neutrinos by pairs of  $\overline{\mathbf{16}}_H$   $SU(5)$ -singlet VEV's which form an effective  $\overline{\mathbf{126}}_H$ . Due to the nature of these VEV's, lepton number is broken but only by one unit, so R-parity is broken. Hence it is necessary to introduce a matter parity in order to preserve the distinction between particles and their super-partners, unlike in the higher-rank Higgs type of models described above where R-parity is automatically preserved.

Vacuum expectation values for the doublets in the  $5(\mathbf{10}_H)$ ,  $\bar{5}(\mathbf{10}_H)$ , and  $\bar{5}(\mathbf{16}_H)$  are then assumed to be generated at the electroweak scale. Since the surviving  $\bar{5} v_d$  VEV is a linear combination of the VEVs of the  $\mathbf{10}_H$  and  $\mathbf{16}_H$ , one finds that  $\tan \beta = v_u/v_d$  can be in the range of 5 - 55 rather than simply 55 when the ratio involves just the two doublets from the  $\mathbf{10}_H$ . While the two doublets in the  $5(\mathbf{10}_H)$  and  $\bar{5}(\mathbf{10}_H)$  Higgs representations contribute in a symmetric way to the  $ij$  components of the Dirac neutrino and up quark mass matrices, and to the charged lepton and down quark mass matrices, respectively, the doublet in the  $\bar{5}(\mathbf{16}_H)$  representation contributes only to the charged lepton and down quark mass matrices in a lopsided fashion. This follows because  $d_L$  and  $\ell_L^c$  lie in a  $10(\mathbf{16})$ , while  $d_L^c$  and  $\ell_L$  lie in a  $\bar{5}(\mathbf{16})$  matter representation. The complete Froggatt-Nielsen tree diagram then makes clear that if a charged lepton  $ij$  mass matrix element

receives a large contribution while the transposed element  $ji$  vanishes, the opposite will be true for the down quark mass matrix. This lopsided behavior for the charged lepton mass matrix can lead to a large lepton flavor violation in  $\mu \rightarrow e + \gamma$ , for example. The corresponding branching ratio for the higher rank Higgs models tends to be one or two orders of magnitude smaller.

In order to obtain a successful  $SO(10)$  GUT model of either type, one must not only be able to generate appropriate neutrino masses and mixings, but the quark masses and CKM mixings for the quark sector must agree with the observed values after evolution downward from the GUT scale. This imposes considerably more constraints on the model than are present with the purely leptonic models discussed in part A. For either type of  $SO(10)$  model, the appearance of a PMNS neutrino mixing matrix close to the tribimaximal mixing form is usually regarded as accidental, rather than reflecting a symmetry inserted at the outset as in the purely leptonic flavor models.

#### 4. $E_6$ and $E_8 \otimes E_8$ Models

Some authors have pursued models based on the exceptional  $E_6$  gauge group. The matter fields of interest are placed in the **27** dimensional representation, while the Higgs fields are placed in a **27<sub>H</sub>**, **351<sub>H</sub>**, and/or **351'<sub>H</sub>** representations. One is then faced with the problem of making massive many of the extra fields which are present in such high dimensional representations. Some progress has been made, but none of the models have any firm numerical predictions for the neutrino mixing angles, so we do not consider them further.

Even more ambitious models have attempted to deal with  $E_8 \otimes E_8$  grand unified models which naturally arise in the heterotic string theory. One of the  $E_8$ 's is assumed to break down to  $E_6 \times SU(3)$ , while the other represents a hidden symmetry. Many of these models, as well as the  $E_6$  models discussed in the previous paragraph, are formulated in five or six dimensions. Again no firm numerical predictions for the mixing angles have been obtained.

## IV. RESULTS FOR THE MODELS SURVEYED

In the previous Sect. we have presented broad general descriptions of models in the categories considered to date. Here we present results for the various models in the literature. We begin by defining our selection criteria for the three neutrino flavor models to be included in our survey. First of all we require that the models give the LMA solution for the solar neutrino oscillations and that firm and reasonably restrictive numerical predictions be given for the reactor  $\sin^2 \theta_{13}$  mixing

parameter. We do not require that the other two mixing angles or the mass squared differences be predicted, but all mixing angles for which information is given are listed in the tables. Models which are clearly in conflict with the present neutrino oscillation data are not considered. Many of the models have evolved with time and have been updated by their authors. As such we have generally listed only the latest published or archived version, except in cases where some important variation has provided two noticeably different results for the reactor angle. Only single references are given in alphabetical order to the accepted models in each category. The interested reader can readily track down earlier references to each model, if they exist, and can use them to learn the specific details of a certain model. Finally, we note that we have arbitrarily selected May 2006 as the cutoff date in accepting models for our compilations.

In Table I we list 26 models cited in [18] and [35] - [54] which exhibit one of the lepton flavor symmetry types: anarchy,  $L_e - L_\mu - L_\tau$ ,  $S_3$  or  $S_4$ ,  $A_4$ ,  $SO(3)$ , and texture zeroes. Only six of them have firm predictions for all three mixing angles. The predictions for  $\sin^2 \theta_{13}$  cover the full range of possibilities, from the present upper bound of 0.025 for the CHOOZ limit down to  $10^{-5}$  or less which would clearly require information from a neutrino factory to measure the result accurately.

In Table II we list seven models cited in [55] - [58] based on sequential right-handed neutrino dominance in which no particular GUT model is assumed. All of these models have a normal hierarchy with a restricted range of predictions for the reactor angle, i.e.,  $\sin^2 \theta_{13} \gtrsim 10^{-3}$ .

In Table III we list 24  $SO(10)$  models cited in [59] - [79] based on Higgs fields in the  $\mathbf{10}_H$ ,  $\overline{\mathbf{126}}_H$ ,  $\mathbf{126}_H$ , and possibly  $\mathbf{120}_H$  or  $\mathbf{45}_H$  dimensional representations. As such, their mass matrix elements receive symmetric or antisymmetric contributions. Note that all models based on a type I seesaw mechanism have normal hierarchy, whereas those with inverted hierarchy would be highly unstable [31]. Only the type II seesaw models permit a stable inverted hierarchy depending upon the interplay of the type I seesaw and left-handed Majorana  $M_{LL}$  contributions. Only three of the models predict a value of  $\sin^2 \theta_{13} \lesssim 10^{-3}$ .

Finally in Table IV are listed six  $SO(10)$  models [80] - [85] involving Higgs fields in the  $\mathbf{10}_H$ ,  $\mathbf{16}_H$ ,  $\overline{\mathbf{16}}_H$ , and  $\mathbf{45}_H$  dimensional representations which exhibit lopsided entries for the down quark and charged lepton mass matrices. Four of them predict values for  $\sin^2 \theta_{13} \lesssim 3 \times 10^{-3}$ . The other two have predictions so near to the CHOOZ bound, they are on the verge of being ruled out.

In order to illustrate these results better visually, we have plotted histograms for the  $\sin^2 \theta_{13}$  predictions. In Fig. 1 we show the results for all 63 models. Only the results for the 34 models which predict all three mixing angles are plotted in Fig. 2. In Figs. 3a and b we separate the models into those with normal and inverted hierarchies, respectively. On the  $\log(\sin^2 \theta_{13})$  scale,

we have divided each power of ten interval into three equal parts. For those models which give a range of values which occupy several intervals, we have rescaled them, so that each model has the same normalized area on each histogram.

Clearly the majority of models prefer normal hierarchy, with the two main exceptions being models based on  $L_e - L_\mu - L_\tau$  symmetry or those with well-designed texture zeros. Roughly half of the 63 models have  $\sin^2 \theta_{13} \gtrsim 0.004$ , or  $\sin^2 2\theta_{13} \gtrsim 0.015$ . This feature remains true even for the smaller number of 34 models which can predict all three mixing angles. Since the next generation of reactor experiments is expected to reach values of order 0.01 for  $\sin^2 2\theta_{13}$ , we can expect that roughly half of the models will be eliminated based on the presence or absence of an observed  $\bar{\nu}_e$  disappearance signal. The possible need for a neutrino factory to reach even smaller values of the reactor neutrino mixing angle, if necessary, will then become apparent. On the other hand, if a disappearance signal is seen, and its value for  $\sin^2 \theta_{13}$  can be well measured, the number of surviving models will be greatly reduced. Determination of the mass hierarchy will narrow the number down even further.

## V. CONCLUSION

From our survey we found that the predictions for the angle  $\theta_{13}$  range from zero to the current experimental upper limit. For models based on GUT symmetries, normal mass hierarchy can be generated naturally. Inverted hierarchy may also be obtained in these models with a type-II seesaw, even though some fine-tuning is needed. Predictions for the mixing angle  $\theta_{13}$  in these models tend to be relatively large, with a median value  $\sin^2 2\theta_{13} \simeq 0.015$ . On the other hand, models based on leptonic symmetries can give rise to inverted mass hierarchy, and the predictions for  $\theta_{13}$  can be quite small. Therefore, if the inverted mass hierarchy is observed experimentally and the mixing angle  $\theta_{13}$  turns out to be tiny, this experimental evidence will then give strong support to models based on lepton symmetries. However, if  $\theta_{13}$  turns out to be relatively large, one will not be able to tell the two different classes apart. A precise measurement for the deviation of  $\theta_{23}$  from  $\pi/4$  can also be crucial for distinguishing different models. This is especially true for models based on lepton symmetries in which the deviation strongly depends on how the symmetry breaking is introduced into the models. Clearly precision measurements are indispensable in order to distinguish different classes of models and to narrow down the number of acceptable models.

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TABLE I: Mixing Angles for Models with Lepton Flavor Symmetry.

Reference	Hierarchy	$\sin^2 2\theta_{23}$	$\tan^2 \theta_{12}$	$\sin^2 \theta_{13}$
<b>Anarchy Model:</b>				
dGM [18]	Either			$\geq 0.011 @ 2\sigma$
<b><math>\mathbf{L}_e - \mathbf{L}_\mu - \mathbf{L}_\tau</math> Models:</b>				
BM [35]	Inverted			0.00029
BCM [36]	Inverted			0.00063
GMN1 [37]	Inverted		$\geq 0.52$	$\leq 0.01$
GL [38]	Inverted			0
PR [39]	Inverted		$\leq 0.58$	$\geq 0.007$
<b><math>\mathbf{S}_3</math> and <math>\mathbf{S}_4</math> Models:</b>				
CFM [40]	Normal			0.00006 - 0.001
HLM [41]	Normal	1.0	0.43	0.0044
	Normal	1.0	0.44	0.0034
KMM [42]	Inverted	1.0		0.000012
MN [43]	Normal			0.0024
MNY [44]	Normal			0.000004 - 0.000036
MPR [45]	Normal			0.01 - 0.064
RS [46]	Inverted	$\theta_{23} \geq 45^\circ$		$\leq 0.02$
	Normal	$\theta_{23} \leq 45^\circ$		0
TY [47]	Inverted	0.93	0.43	0.0025
T [48]	Normal			0.0016 - 0.0036
<b><math>\mathbf{A}_4</math> Tetrahedral Models:</b>				
ABGMP [49]	Normal	0.997 - 1.0	0.365 - 0.438	0.0037 - 0.00069
AKKL [50]	Normal			0.04 - 0.006
Ma [51]	Normal	1.0	0.45	0
<b><math>\mathbf{SO}(3)</math> Models:</b>				
M [52]	Normal	0.87 - 1.0	0.46	0.00005
<b>Texture Zero Models:</b>				
CPP [53]	Normal			0.081 - 0.091
	Inverted			$\geq 0.007$
	Inverted			$\geq 0.032$
WY [54]	Either			0.0006 - 0.003
	Either			0.002 - 0.02
	Either			0.02 - 0.15

TABLE II: Mixing Angles for Models with Sequential Right-Handed Neutrino Dominance.

Reference	Hierarchy	$\sin^2 2\theta_{23}$	$\tan^2 \theta_{12}$	$\sin^2 \theta_{13}$
D [55]	Normal			0.008 - 0.14
EH [56]	Normal	0.98	0.32	0.014
	Normal	0.98	0.34	0.012
	Normal	0.99	0.45	0.0009
	Normal	0.97	0.30	0.014
H [57]	Normal	1.0	0.42	0.0033
K [58]	Normal	0.99 - 1.0	0.40 - 0.62	0.0027

 TABLE III: Mixing Angles for  $SO(10)$  Models with Symmetric/Antisymmetric Contributions.

Reference	Hierarchy	$\sin^2 2\theta_{23}$	$\tan^2 \theta_{12}$	$\sin^2 \theta_{13}$
BaMa [59]	Normal	0.88	0.33	0.015 - 0.028
	Normal	0.98	0.44	0.013
	Inverted	0.88	0.29	0.024
BMSV [60]	Inverted			$\geq 0.01$
BKOT [61]	Normal	0.98	0.28	0.0001 - 0.0006
BO [62]	Normal	0.98 - 1.0	0.29 - 0.46	0.0014
BN [63]	Normal			0.0009 - 0.016
BeMa [64]	Normal	0.93	0.40	0.012
BRT [65]	Normal	0.99	0.35	0.0024
BW [66]	Normal			O(0.01)
CM [67]	Normal	1.0	0.41	0.014
DR [68]	Normal	0.98	0.40	0.0025
DMM [69]	Normal			0.0036 - 0.012
FO [70]	Normal	0.90	0.31	0.04
GMN2 [71]	Normal	$\leq 0.91$	$\geq 0.52$	0.026
KR [72]	Normal	0.93	0.44	0.058
O [73]	Normal	0.94	0.46	0.0007
Ra [74]	Normal			O(0.01)
Ro [75]	Normal			0.0056
	Inverted			0.036
ST [76]	Normal	0.99	0.46	0.0001 - 0.04
SP [77]	Normal	0.99	0.42	0.0002
VR [78]	Normal	0.99 - 1.0	0.40 - 0.61	0.024
YW [79]	Normal	0.96	0.40	0.04

TABLE IV: Mixing Angles for **SO(10)** Models with Lopsided Mass Matrices.

Reference	Hierarchy	$\sin^2 2\theta_{23}$	$\tan^2 \theta_{12}$	$\sin^2 \theta_{13}$
A [80]	Normal	0.98 - 1.0	0.38 - 0.50	0.002 - 0.003
AB [81]	Normal	0.99	0.49	0.0002
BB [82]	Normal	0.97	0.40	0.0016 - 0.0025
JLM [83]	Normal	1.0	0.41	0.019
Mae [84]	Normal			0.048
P [85]	Normal	0.99	0.17 - 0.29	0.0004 - 0.0025

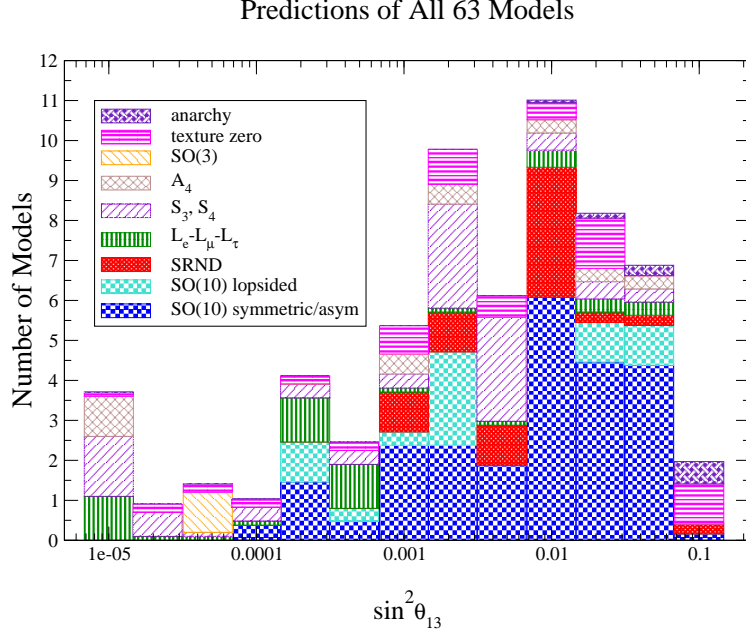


FIG. 1: Histogram of the number of models for each  $\sin^2 \theta_{13}$  including all 63 models.

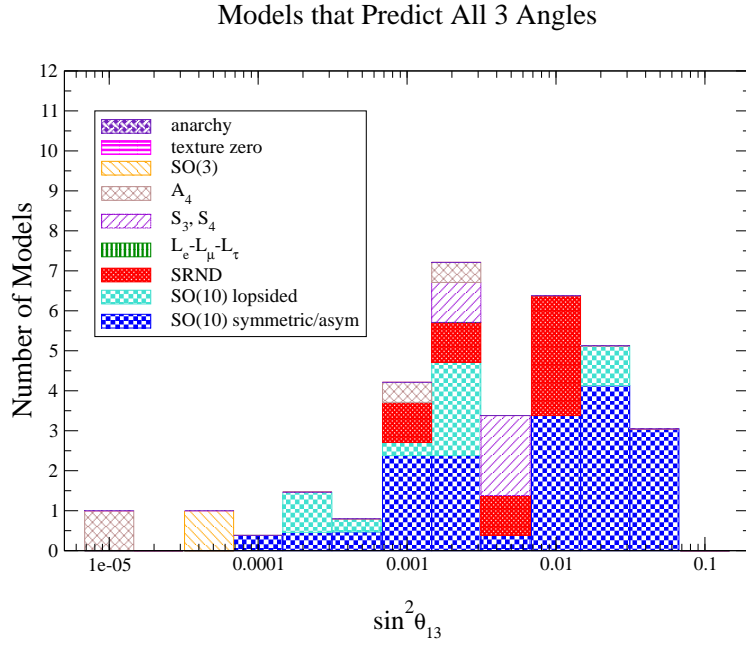


FIG. 2: Histogram of the number of models for each  $\sin^2 \theta_{13}$  that give predictions for all three leptonic mixing angles.

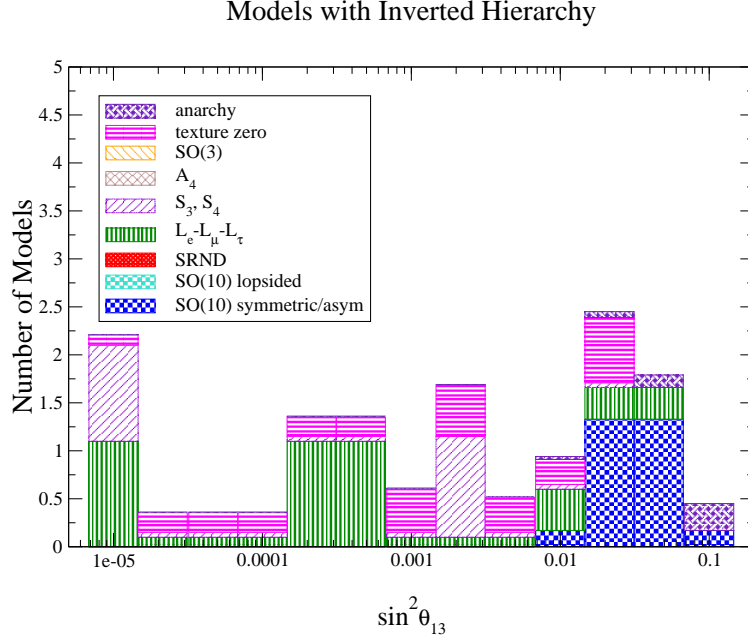
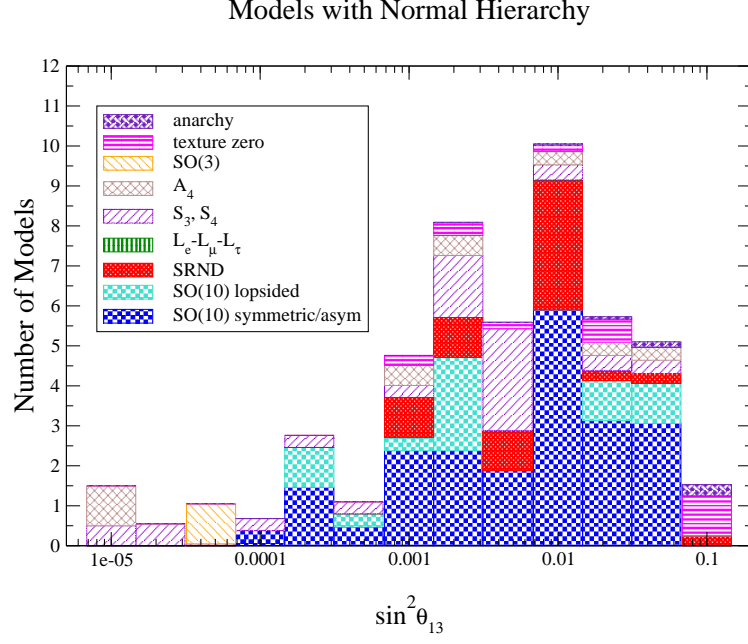


FIG. 3: Histograms of the number of models for each  $\sin^2 \theta_{13}$  where the upper diagram includes models that predict normal mass hierarchy, while the lower diagram includes models that predict inverted mass hierarchy.